Generative Adversarial Networks

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October 10, 2018

Review:

We have two neural networks competing

- Generator: $G : \mathcal{H} \to \mathcal{X}$.
- **Discriminator**: $D : \mathcal{X} \rightarrow [0, 1]$

We want to find the parameters that reach equilibrium for the minmax game

$$G^* = \operatorname*{argmin}_{G} \max_{D} \frac{1}{2} \left(\mathbb{E}[\log D(X)] + \mathbb{E}[\log (1 - D(X_G))] \right). \tag{1}$$

where $X_G = G(H)$ are the "fake" samples, given by the distribution induced by *G* from p_H .

Review:



Game Saturation:

- The cost for the generator function $\mathbb{E}[\log(1 D(X_G))]$ is useful in theory, but performs badly in practice.
- The "fake" samples can be so "unrealistic" in the beginning, that the response of *D* saturates.
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• Instead we modify the cost when updating θ_{G}

$$c_G = -\frac{1}{2} \mathbb{E}[\log\left(D(X_G)\right)] \tag{2}$$

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Deep Convolutional GANs (DCGANs)

Reference: Radford et al. 2015, Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks.



• PyTorch has DCGAN in his set of examples:

https://github.com/pytorch/examples.git

Deep Convolutional GANs (DCGANs)

The authors did an extensive model exploration and identified a set of rules that resulted in more stable training:

- Only convolutional layers, no fully-connected.
- Instead of pooling layers, use strided convolutions for *D* and strided transpose convolutions for *G*.
- Use batchnorm on both G and D.
- Use ReLu in G except for the output, where it uses Tanh.
- Use the LeakyReLu for all D.

colab notebook

This version presents better stability, ans prevents "model collapse".

• The original optimization problem in (1) is equivalent to

$$G^* = \operatorname{argmin}_{G} D_{JS}(p_{data}, p_G)$$
(3)

where D_{JS} is the **Jensen-Shannon Divergent**, a standard dissimilarity measure on the space of distributions.

- Nevertheless, *D*_{JS} doesn't really capture a metric structure on the space of distributions.
- The alternative choice is to use the Wasserstein metric.

· The Wasserstein distance we'll use is defined by

$$W(\mu,\nu) = \inf_{\eta \in \Pi(\mu,\nu)} \mathbb{E}_{(X,\tilde{X}) \sim \eta} \left[\| X - \tilde{X} \| \right]$$
(4)

where $\Pi(\mu, \nu)$ are distributions with marginals μ and ν (coupling).

• This distribution is sometimes known as the *"earth moving distance"*, which gives the interpretation of the minimum mass one has to move to transform one distribution into the other.

• The idea would be finding a generator matching on this metric

$$G^* = \underset{G}{\operatorname{argmin}} W(p_{data}, p_G) \tag{5}$$

which is unfortunately unfeasible.

• We use instead an equivalent form, given by a result of Kantorovich and Rubinstein

$$W(\mu,\nu) = \max_{\|f\|_{\iota} \le 1} \mathbb{E}_{\substack{X \sim \mu \\ \tilde{X} \sim \nu}} \left[f(X) - f(\tilde{X}) \right]$$
(6)

where $\|f\|_{L} = \sup_{x,\tilde{x}} \frac{\|f(x) - f(\tilde{x})\|}{\|x - \tilde{x}\|}$ is the Lipschitz seminorm.

· Such formulation allows to formulate the problem as

$$G^* = \underset{G}{\operatorname{argmin}} W(p_{data}, p_G)$$

=
$$\underset{G}{\operatorname{argmin}} \max_{\|D\|_{L} \le 1} \left(\mathbb{E}_{X \sim p_{data}} \left[D(X) \right] - \mathbb{E}_{X_G \sim p_G} \left[D(X_G) \right] \right)$$
(7)

• The main difficulty here is to be able to optimize *D* while restricted to the condition $||D||_{l} \le 1$.

- The original way of training is by clipping the weights of D.
- The alternative is known as WGAN-GP, because it adds a gradient penalty, a smooth replace for the original discriminator search

$$D^* = \operatorname*{argmax}_{D} \mathbb{E}_{X \sim p_{\text{data}}}[D(X)] - \mathbb{E}_{X_G \sim p_G}[D(X_G)] - \lambda \mathbb{E}_{X_u \sim p_u}[(\|\nabla D(X_u)\| - 1)^2]$$
(8)

where p_u is a uniformly sample between a real (p_{data}) and a fake one (p_G) . That is $X_u = UX + (1 - U)X_G$ for $U \sim U[0, 1]$.

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, c = 0.01, m = 64, $n_{\text{critic}} = 5$.

Require: : α , the learning rate. c, the clipping parameter. m, the batch size. n_{critic} , the number of iterations of the critic per generator iteration.

Require: : w_0 , initial critic parameters. θ_0 , initial generator's parameters.

1: while θ has not converged do

2: **for**
$$t = 0, ..., n_{\text{critic}}$$
 do

3: Sample $\{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r$ a batch from the real data.

4: Sample
$$\{z^{(i)}\}_{i=1}^m \sim p(z)$$
 a batch of prior samples.

5:
$$g_w \leftarrow \nabla_w \left[\frac{1}{m} \sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)})) \right]$$

6:
$$w \leftarrow w + \alpha \cdot \operatorname{RMSProp}(w, g_w)$$

7:
$$w \leftarrow \operatorname{clip}(w, -c, c)$$

8: end for

9: Sample
$$\{z^{(i)}\}_{i=1}^m \sim p(z)$$
 a batch of prior samples.

10:
$$g_{\theta} \leftarrow -\nabla_{\theta} \frac{1}{m} \sum_{i=1}^{m} f_w(g_{\theta}(z^{(i)}))$$

11: $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_{\theta})$

12: end while

Algorithm 1 WGAN with gradient penalty. We use default values of $\lambda = 10$, $n_{\text{critic}} = 0.0001$, $\beta_1 = 0$, $\beta_2 = 0.9$.

Require: The gradient penalty coefficient λ , the number of critic iterations per generator i n_{critic} , the batch size *m*, Adam hyperparameters α , β_1 , β_2 .

Require: initial critic parameters w_0 , initial generator parameters θ_0 .

- 1: while θ has not converged do
- for $t = 1, ..., n_{\text{critic}}$ do 2: 3: for i = 1, ..., m do Sample real data $\boldsymbol{x} \sim \mathbb{P}_r$, latent variable $\boldsymbol{z} \sim p(\boldsymbol{z})$, a random number $\epsilon \sim U$ 4: 5: $\tilde{\boldsymbol{x}} \leftarrow G_{\boldsymbol{\theta}}(\boldsymbol{z})$ $\hat{\boldsymbol{x}} \leftarrow \epsilon \boldsymbol{x} + (1-\epsilon) \tilde{\boldsymbol{x}}$ 6: $L^{(i)} \leftarrow D_w(\tilde{\boldsymbol{x}}) - D_w(\boldsymbol{x}) + \lambda (\|\nabla_{\hat{\boldsymbol{x}}} D_w(\hat{\boldsymbol{x}})\|_2 - 1)^2$ 7: 8: end for $w \leftarrow \operatorname{Adam}(\nabla_w \frac{1}{m} \sum_{i=1}^m L^{(i)}, w, \alpha, \beta_1, \beta_2)$ 9: 10: end for Sample a batch of latent variables $\{z^{(i)}\}_{i=1}^m \sim p(z)$. 11: $\theta \leftarrow \operatorname{Adam}(\nabla_{\theta} \frac{1}{m} \sum_{i=1}^{m} -D_w(G_{\theta}(\boldsymbol{z})), \theta, \alpha, \beta_1, \beta_2)$ 12:
- 13: end while

Tips and Tricks

Train with labels:

- When available, using labels in any way in most cases results in a dramatic improvement in the quality of the samples.
- This was observed while the community explored ways of class-conditioning the distributions.
- Perhaps, the reason for improvement is that giving extra information facilitates optimization during training.
- Classes also point out information that we humans tend to focus, and therefore the model can develop a bias that pleases our perspective.
- Sometimes just by training the model class by class also helps convergence.

Tips and Tricks

Feature Matching:

• The idea is to improve statistical difference in the generator cost

$$\|\mathbb{E}_{x \sim p_{\text{data}}} f(X) - \mathbb{E}_{x_{C} \sim p_{G}} f(X_{G})\|_{2}^{2}$$
(9)

using an intermediate feature response instead of D(x).



Tips and Tricks

Discriminator and generator network capacity

- Usually the discriminator is more complex than the generator. The opposite can even make the game diverge.
- When doing model selecting, starting from your base model.
 - Adding more neurons in a layer, usually doesnt harm the model, and convergence is maintained.
 - Adding more layers can go on the opposite direction, of actually harming performance. (This might change if your architecture has residual layer for example)
 - Is important to identify the bottleneck for your application.